Supplementary Material to "Understanding Networks with Exponential-family Random Network Models"

Guide

In this file we give the derivation of parameter interpretation of ERNM in section 1 and 2. We show the goodness-of-fit plots for both the ERGM and ERNM fits to the adolescent health network data in Section 3, and MCMC diagnostics results in Section 4.

1. ERNM Logit Form for Edge Connections

$$\begin{aligned}
\mathbb{P} &\equiv P_\eta(y_{ij} = 1 | x, y^c_{ij}) \\
&= \frac{P_\eta(y_{ij} = 1, x, y^c_{ij})}{P_\eta(x, y^c_{ij})} \\
&= \frac{P_\eta(y_{ij} = 1, x, y^c_{ij})}{P_\eta(y_{ij} = 1, x, y^c_{ij}) + P_\eta(y_{ij} = 0, x, y^c_{ij})} \\
&= \frac{\exp \eta \cdot g(y^-_{ij}, y_{ij} = 1, x)}{\exp\{\eta \cdot g(y^-_{ij}, y_{ij} = 1, x)\} + \exp\{\eta \cdot g(y^-_{ij}, y_{ij} = 0, x)\}} \\
&= \frac{1}{1 + \exp\left\{\eta \cdot \left(g(y^-_{ij}, x) - g(y^+_{ij}, x)\right)\right\}},
\end{aligned} \tag{1}$$

$$\begin{aligned}
\text{logit}\mathbb{P} &= \log \frac{\mathbb{P}}{1 - \mathbb{P}} \\
&= \log \frac{\frac{1}{1+\exp\left\{\eta \cdot \left(g(y^-_{ij}, x) - g(y^+_{ij}, x)\right)\right\}}}{1 - \frac{1}{1+\exp\left\{\eta \cdot \left(g(y^-_{ij}, x) - g(y^+_{ij}, x)\right)\right\}}} \\
&= \log \exp\left\{-\eta \cdot \left(g(y^-_{ij}, x) - g(y^+_{ij}, x)\right)\right\} \\
&= \eta \cdot \left(g(y^+_{ij}, x) - g(y^-_{ij}, x)\right).
\end{aligned} \tag{2}$$

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2. ERNM Logit Form for Dyadic Attributes

$$
\begin{aligned}
\mathbb{P} &\equiv P_\eta(z_{ij}=1|z_{ij}^c, t_{ij}, Y=y) \\
&= \frac{P_\eta(z_{ij}=1, z_{ij}^c, t_{ij}, Y=y)}{P_\eta(z_{ij}^c, t_{ij}, Y=y)} \\
&= \frac{P_\eta(z_{ij}=1, z_{ij}^c, t_{ij}, Y=y)}{P_\eta(z_{ij}=0, z_{ij}^c, t_{ij}, Y=y) + P_\eta(z_{ij}=1, z_{ij}^c, t_{ij}, Y=y)} \\
&= \frac{\exp\left\{t_{ij}\beta + \alpha g(y, t_{ij}) + \lambda(h(y, z_{ij}=1) + h(y, z_{ij}^c))\right\}}{\exp\left\{t_{ij}\beta + \alpha g(y, t_{ij}) + \lambda(h(y, z_{ij}=0) + h(y, z_{ij}^c))\right\} + \exp\left\{\alpha g(y, t_{ij}) + \lambda(h(y, z_{ij}=1) + h(y, z_{ij}^c))\right\}} \\
&= \frac{\exp\left\{t_{ij}\beta\right\}}{\exp\left\{t_{ij}\beta\right\} + \exp\left\{\lambda(h(y, z_{ij}=0) - h(y, z_{ij}=1)\right\}} \\
&= \frac{1}{1+\exp\left\{\lambda(h(y, z_{ij}^-) - h(y, z_{ij}^+)) - t_{ij}\beta\right\}},
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\text{logit}\mathbb{P} &= \log\frac{\mathbb{P}}{1-\mathbb{P}} \\
&= \log\frac{\frac{1}{1+\exp\left\{\lambda(h(y,z_{ij}^-)-h(y,z_{ij}^+))-t_{ij}\beta\right\}}}{1-\frac{1}{1+\exp\left\{\lambda(h(y,z_{ij}^-)-h(y,z_{ij}^+))-t_{ij}\beta\right\}}} \\
&= \log\exp\left\{-\left(\lambda(h(y, z_{ij}^-) - h(y, z_{ij}^+)) - t_{ij}\beta\right)\right\} \\
&= (t_{ij}\beta) - \left(\lambda(h(y, z_{ij}^-) - h(y, z_{ij}^+))\right).
\end{aligned} \tag{4}
$$

3. Goodness of Fit

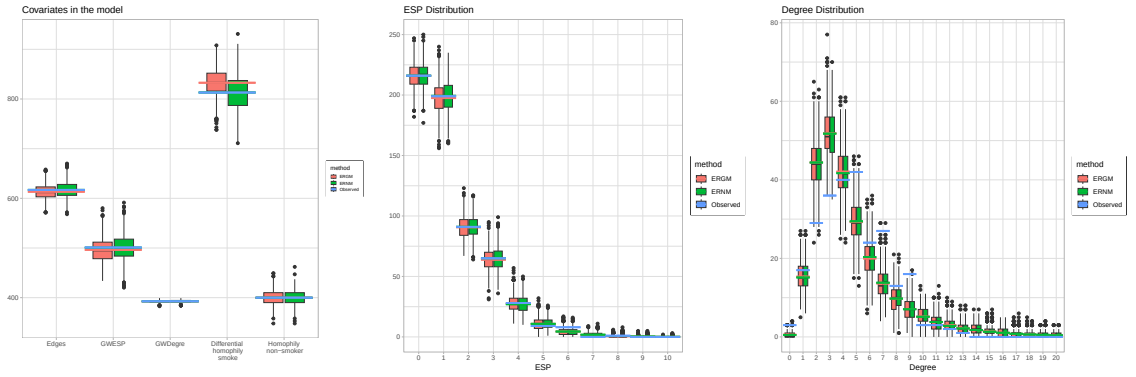

Figure 1: GOF Comparison of ERGM and ERNM (Model 1): Grade 9

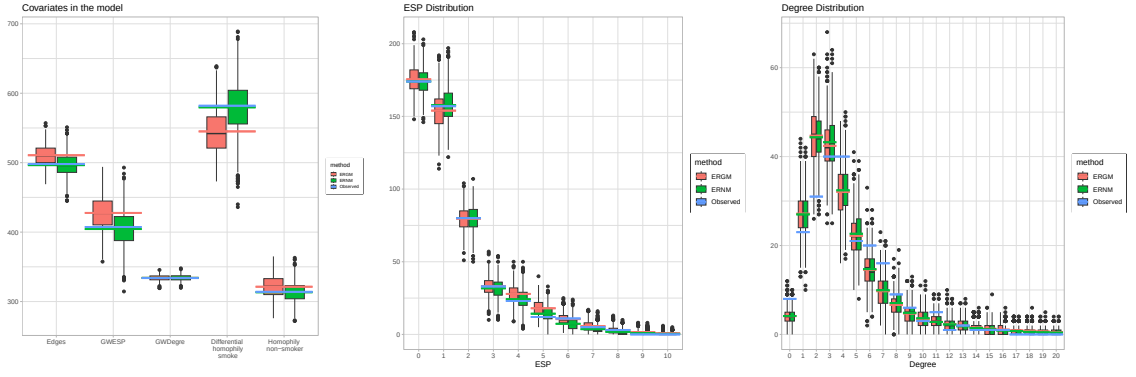

Figure 2: GOF Comparison of ERGM and ERNM (Model 1): Grade 10

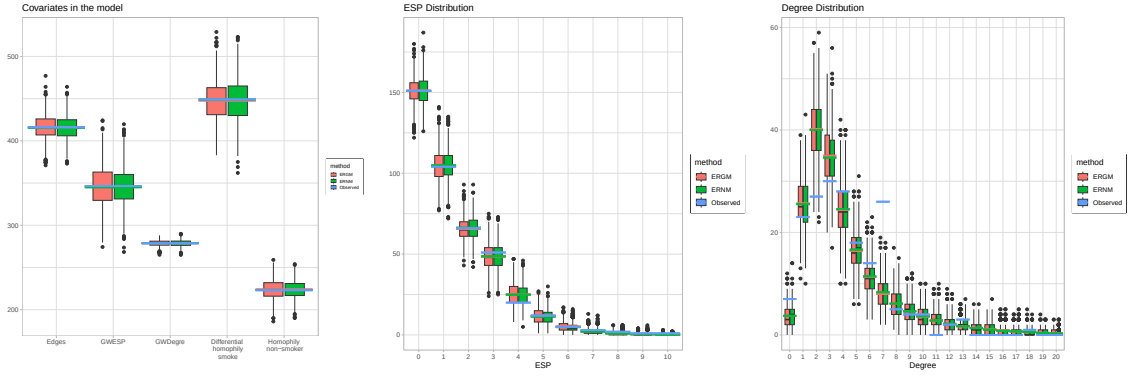

Figure 3: GOF Comparison of ERGM and ERNM (Model 1): Grade 11

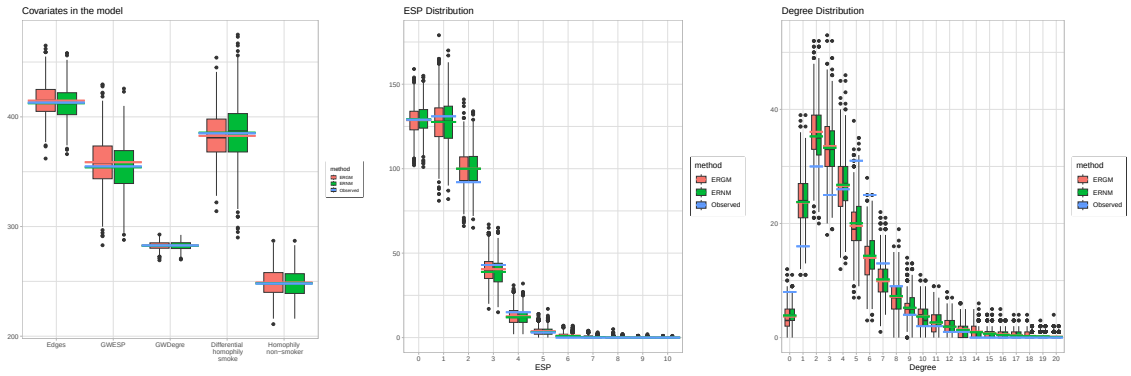

Figure 4: GOF Comparison of ERGM and ERNM (Model 1): Grade 12

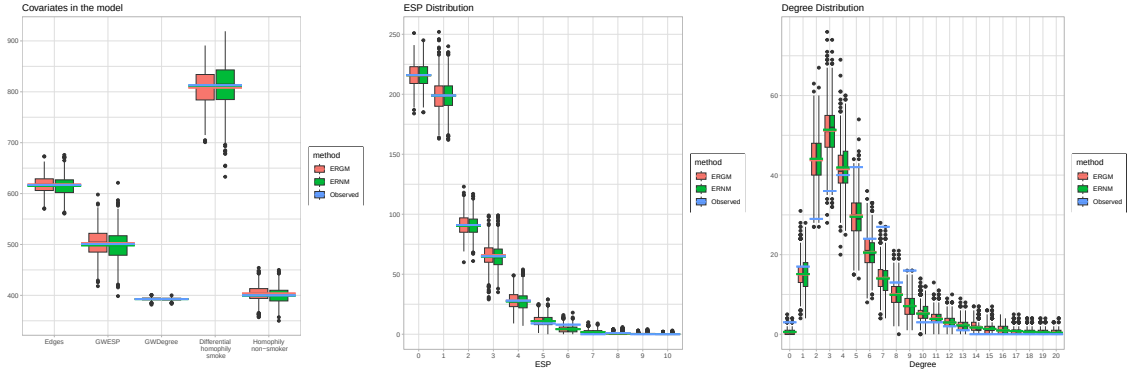

Figure 5: GOF Comparison of ERGM and ERNM-Count (Model 2): Grade 9

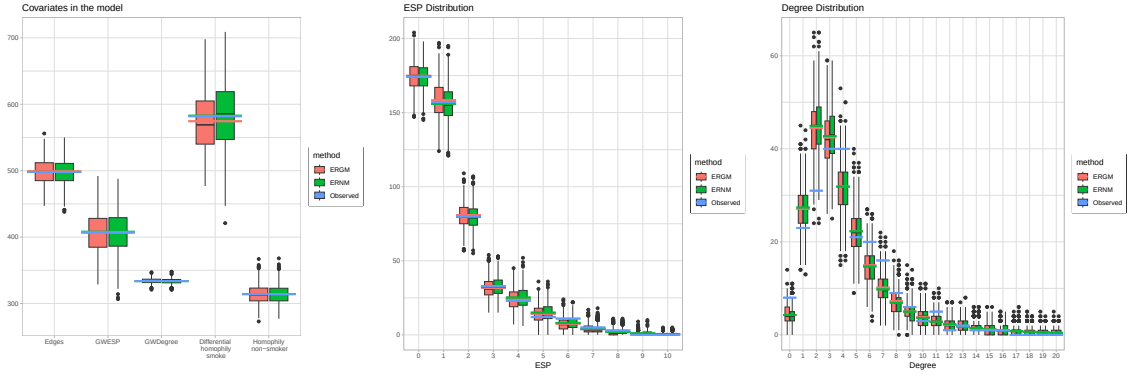

Figure 6: GOF Comparison of ERGM and ERNM-Count (Model 2): Grade 10

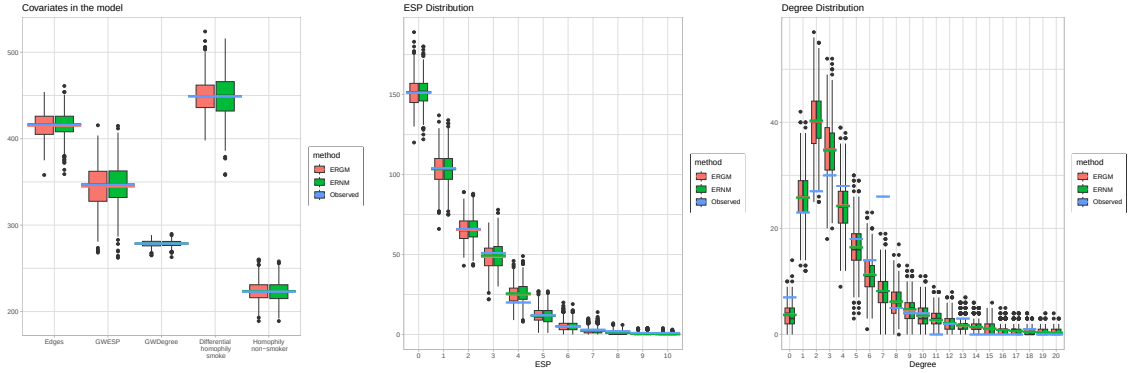

Figure 7: GOF Comparison of ERGM and ERNM-Count (Model 2): Grade 11

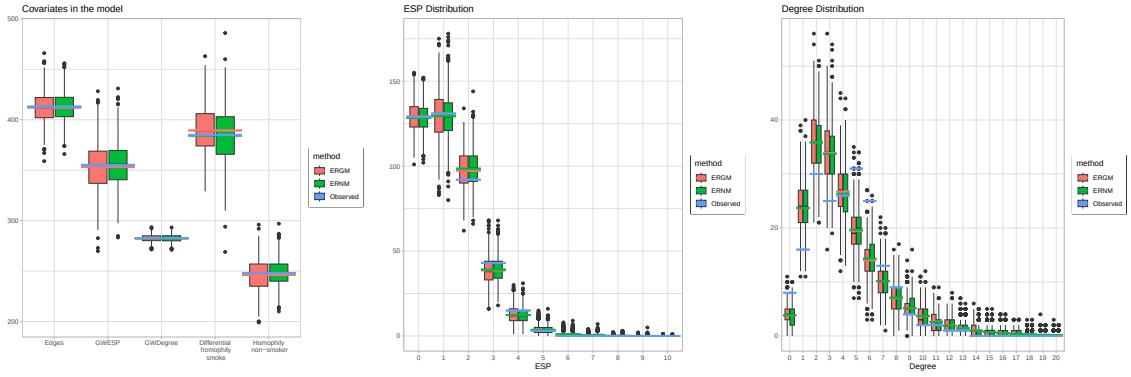

Figure 8: GOF Comparison of ERGM and ERNM-Count (Model 2): Grade 12

4. MCMC Diagnostics

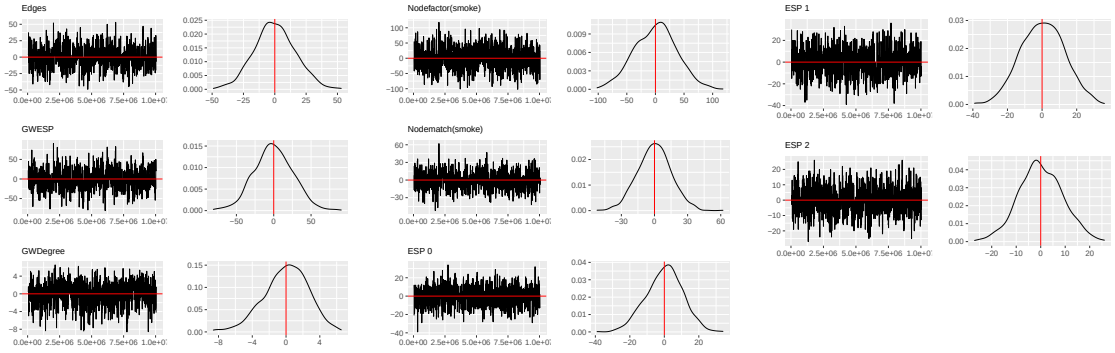

Figure 9: MCMC Diagnostics of ERNM (Model 1) for grade 9: Left Column: Trace plot; Right Column: Density plot

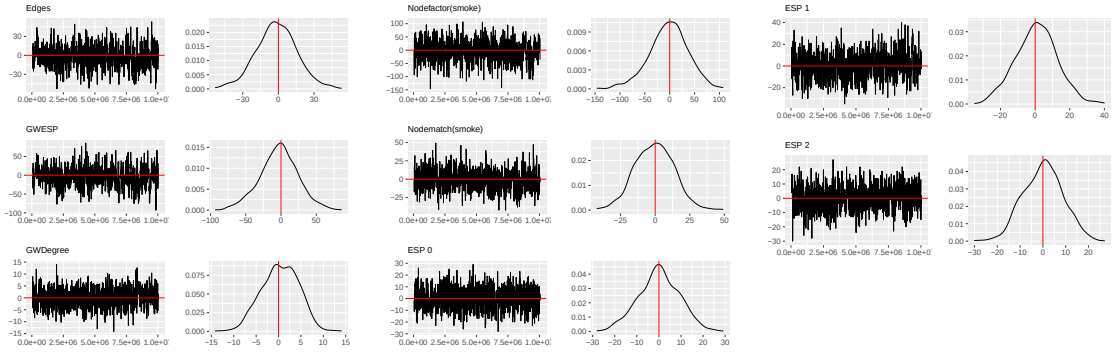

Figure 10: MCMC Diagnostics of ERNM (Model 1) for grade 10: Left Column: Trace plot; Right Column: Density plot

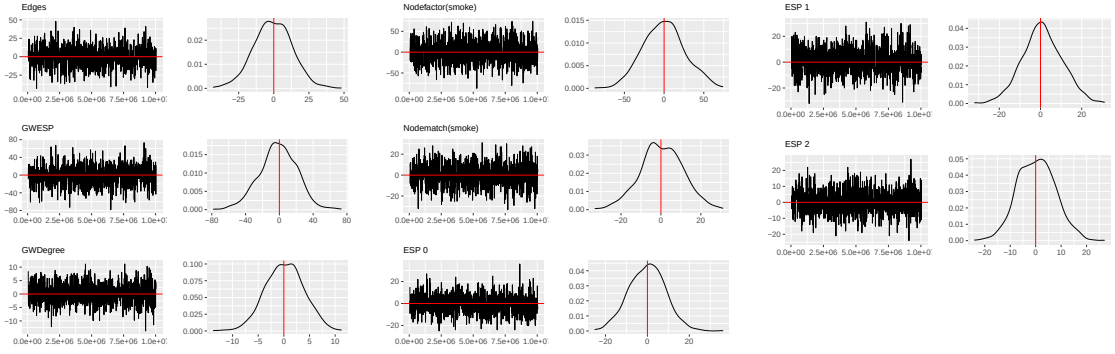

Figure 11: MCMC Diagnostics of ERNM (Model 1) for grade 11: Left Column: Trace plot; Right Column: Density plot

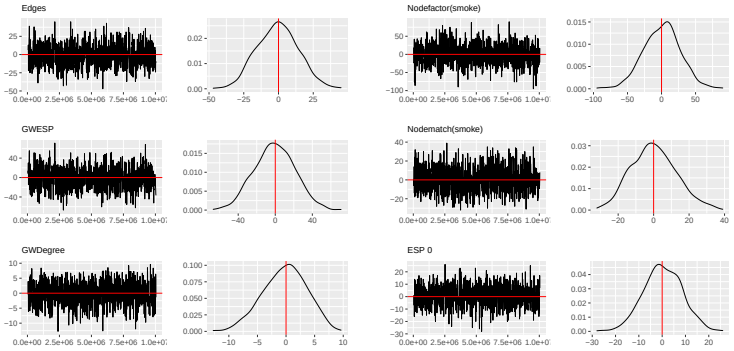

Figure 12: MCMC Diagnostics of ERNM (Model 1) for grade 12: Left Column: Trace plot; Right Column: Density plot

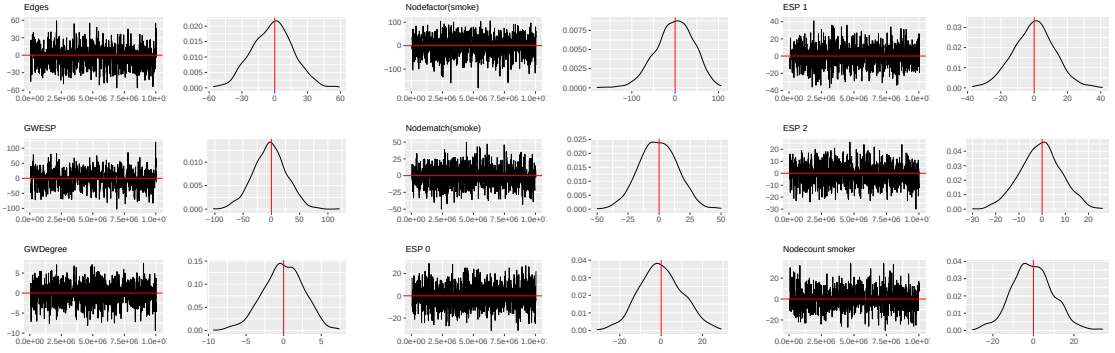

Figure 13: MCMC Diagnostics of ERNM-Count (Model 2) for grade 9: Left Column: Trace plot; Right Column: Density plot

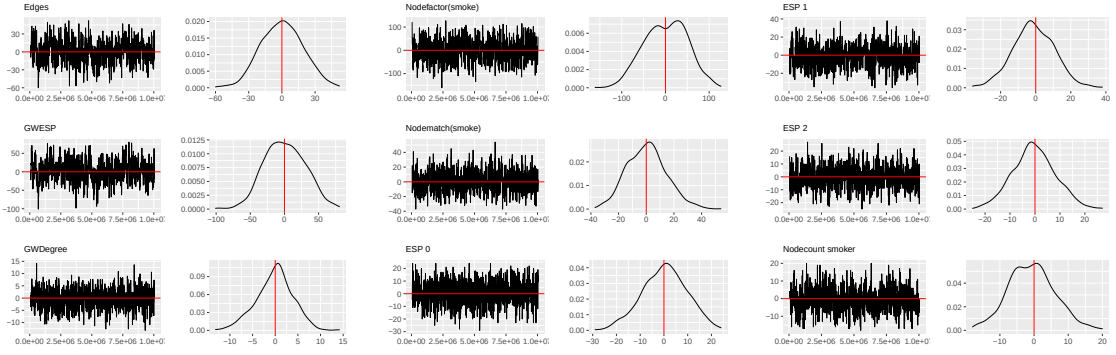

Figure 14: MCMC Diagnostics of ERNM-Count (Model 2) for grade 10: Left Column: Trace plot; Right Column: Density plot

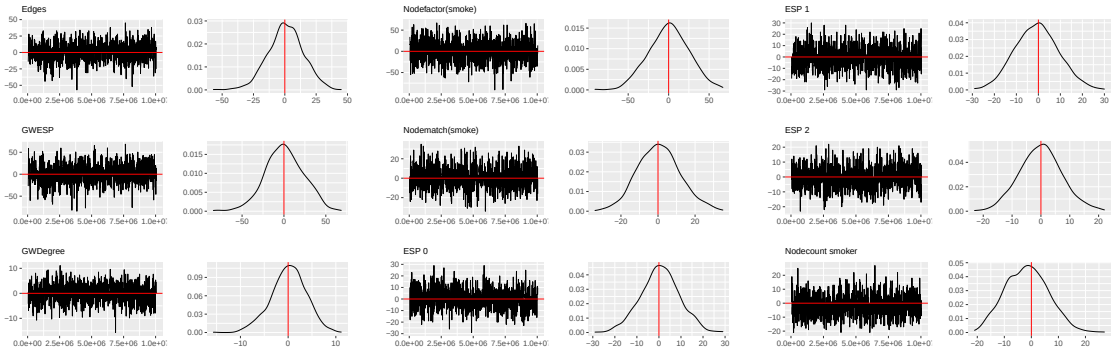

Figure 15: MCMC Diagnostics of ERNM-Count (Model 2) for grade 11: Left Column: Trace plot; Right Column: Density plot

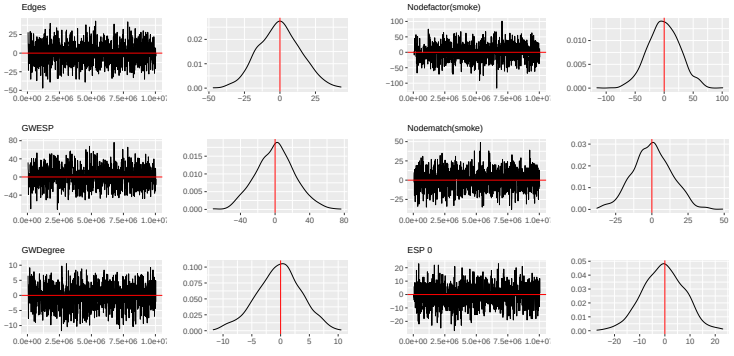

Figure 16: MCMC Diagnostics of ERNM-Count (Model 2) for grade 12: Left Column: Trace plot; Right Column: Density plot

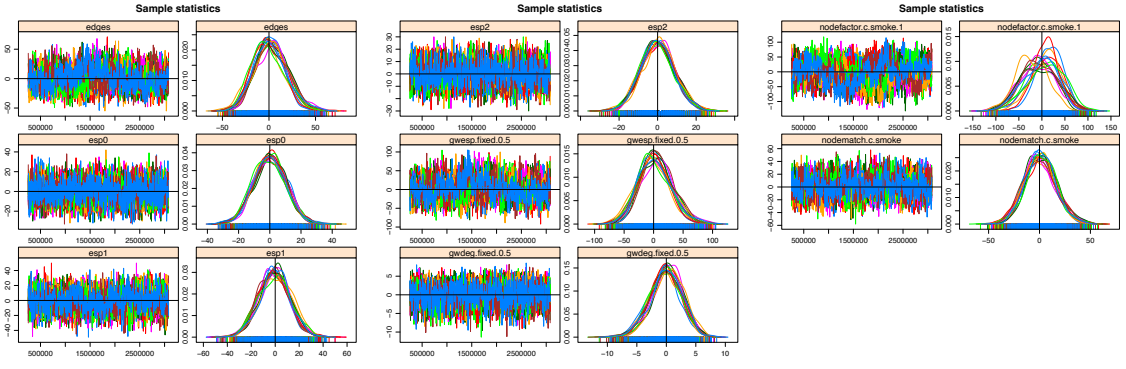

Figure 17: MCMC Diagnostics of ERGM for Grade 9: Left Column: Trace plot; Right Column: Density plot. (The line with different color represents each parallelled Markov chain)

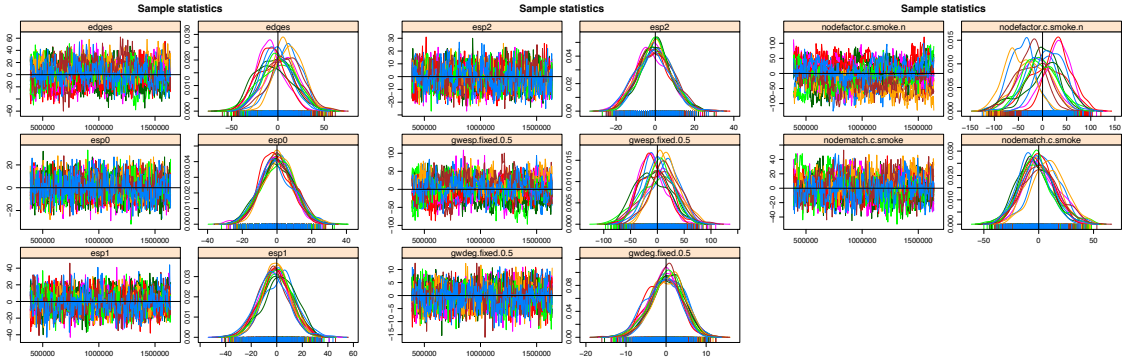

Figure 18: MCMC Diagnostics of ERGM for Grade 10: Left Column: Trace plot; Right Column: Density plot. (The line with different color represents each parallelled Markov chain)

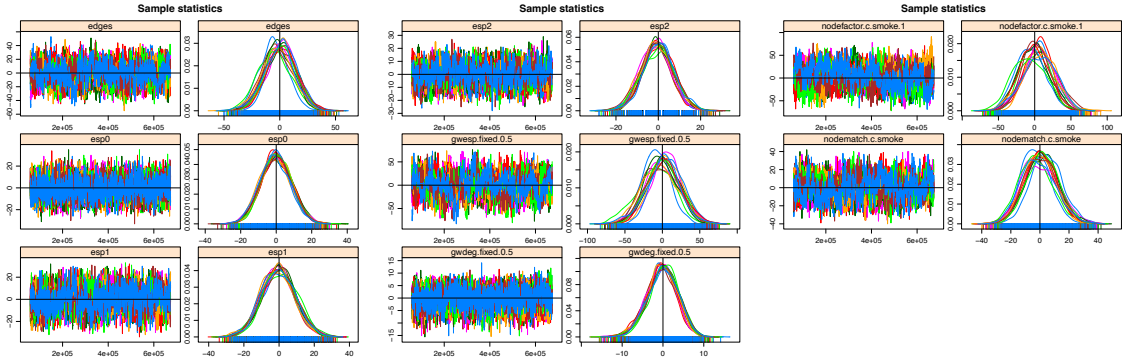

Figure 19: MCMC Diagnostics of ERGM for Grade 11: Left Column: Trace plot; Right Column: Density plot. (The line with different color represents each parallelled Markov chain)

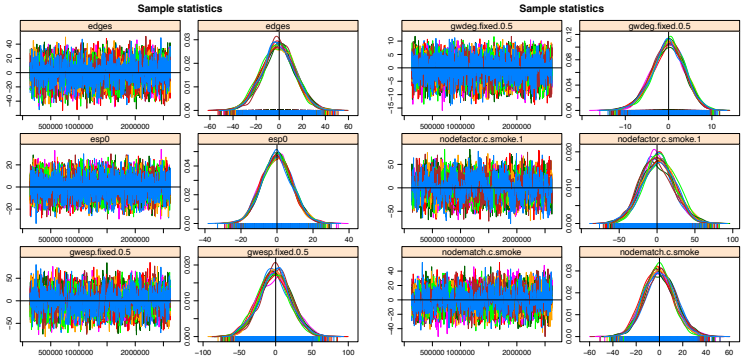

Figure 20: MCMC Diagnostics of ERGM for Grade 12: Left Column: Trace plot; Right Column: Density plot. (The line with different color represents each parallelled Markov chain)