reasonably sharp bend in the curve around $x = 1750$. The bending seems also to extend for some distance to the right. Furthermore, this bending is accompanied by an elevated level of variance. Could this be extra variation introduced as the new metabolic pathway kicks in? There is also an outlier at $x = 3452$, whose cause needs to be investigated.

This highly speculative analysis suggests an anaerobic threshold of around 1750. Without the lag, the estimate could well be shifted to the right by about 11 values of x_i, i.e., to about 2500. Thus the lag could go a long way toward explaining the previously reported bias. Could the incorporation of lags lead to an improved method for estimating anaerobic threshold?

REFERENCES

Bennett, G.W. (1988). Determination of anaerobic threshold. *Canad. J. Statist.*, 16, 307–316.

deVries, H.A. (1986). *Physiology of Exercise for Physical Education and Athletes*. Fourth Edition. Wm. C. Brown, Dubuque, Iowa.

Green, H.J., Hughson, R.L., Orr, G.W., and Ranney, D.A. (1983). Anaerobic threshold, blood lactate, and muscle metabolites in progressive exercise. *J. Appl. Physiol. Respirat. Environ. Exercise Physiol.*, 54, 1032–1038.

Wasserman, K., Whipp, B.J., Koyal, S.N., and Beaver, W.L. (1973). Anaerobic threshold and respiratory gas exchange during exercise. *J. Appl. Physiol.*, 35, 236–243.

Yeh, M.P., Gardner, R.M., Adams, T.D., Yanowitz, F.G., and Crapo, R.O. (1983). "Anaerobic threshold": Problems of determination and validation. *J. Appl. Physiol. Respirat. Environ. Exercise Physiol.*, 55, 1178–1186.

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Determination of Anaerobic Threshold: What Anaerobic Threshold?

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In a recent article in the Consultant's Corner section of this journal, Bennett (1988) considers a problem in the area of kinesiology. Apparently, when humans exercise strenuously, an "anaerobic threshold" is passed; this threshold is defined to be the level of oxygen consumption "just below that at which an abrupt change in metabolism occurs accompanied by associated changes in the exchange of gases in the lungs". This is thought to be reflected in the relationship of expired ventilation (y) to oxygen uptake (x). For the particular data set, pertaining to a single individual taking part in one particular test, given in Table 1 of Bennett (1988), we can see no evidence of any sudden change of the kind expected. While we are not qualified to doubt the existence of the anaerobic threshold in general, we feel we must point out that it appears that either (a) an abrupt change in metabolic activity does not necessarily translate directly to a similar change in expired ventilation, or (b) at least for some individuals, the physiological changes take place more gradually.

The first, and most convincing, evidence for our contention comes from the data plot

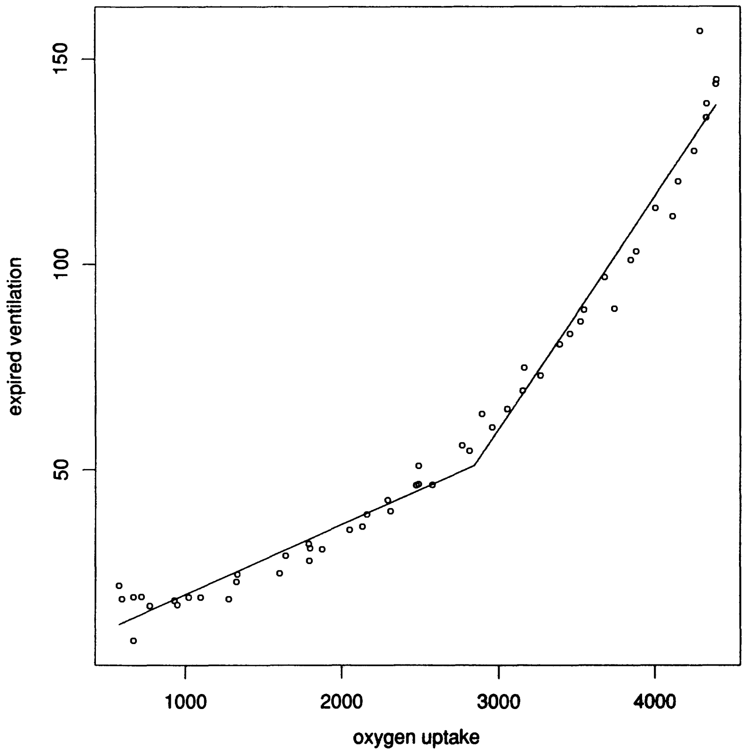

FIGURE 2: The data and the fitted two-segment linear function of Bennett (1988).

given in Figure 1 of Bennett (1988) itself. We find it difficult to conceive of anyone fitting a two-piece linear (as Bennett does), or similar abruptly changing, functional form to these data without a very strong prior conviction that such is appropriate (incidentally, we have no comments to pass on the fitting algorithm for the piecewise linear function, a description of which fills the bulk of Bennett's paper). To emphasize our point, in our Figure 2 we display the data again but with the best two-segment linear fit obtained by Bennett superimposed (details are for the "unconstrained" case in Bennett's Appendix 2).

It is immediately apparent that this is an inadequate fit to the data, as we observe a systematic pattern of departures from the function, which is equally clear on a plot of residuals from this fit (not shown).

As further illustration, in Figure 3 we present the result of fitting (by least squares) a more natural functional form — a quadratic — empirically motivated by Figure 1 of Bennett (1988). To get Figure 3, we deleted two points (the third and forty-ninth of Bennett's Table 1), which are somewhat outlying on Figure 2 and which may be the result of misprinted ventilation values in Bennett's Table 1 (certainly the column headings there have been reversed). Without those two points, a similar two-segment linear fit to the data of Figure 2 is obtained with a residual sum of squares (RSS) of 986. The quadratic fit of Figure 3 — which employs one parameter less than the piecewise linear — reduces the RSS all the way to 489. The fitted quadratic is

$$\hat{y} = 53.3 + 0.035(x - 2713) + 9.0 \times 10^{-6}(x - 2713)^2. \qquad (1)$$

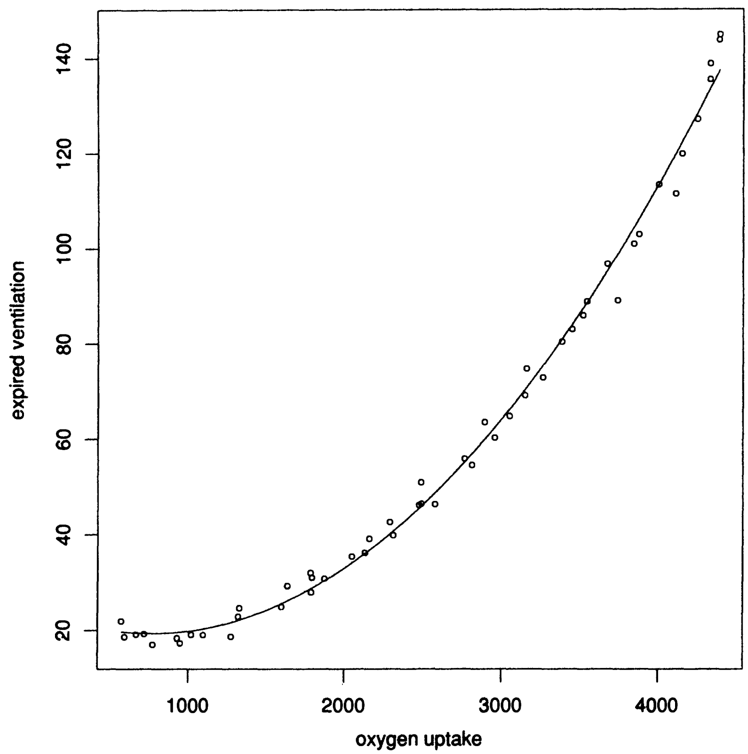

FIGURE 3: The quadratic function (1) superimposed on the data with two outlying points deleted.

The quadratic fit to the original data provides a similar curve with an RSS of 1193 as against 1661 for the two-piece linear model. Now, we are not claiming that (1) is a definitive fit to this data set; indeed, there remains scope for a variety of further investigations which could improve on our empirical curve fitting. Rather, we wish to use the improved fit attained by the quadratic to demonstrate that, for this data set, a better explanation of the relationship between y and x is afforded by a smoothly varying function. Thus, we cannot quarrel with the notion that expired ventilation increases monotonically with oxygen uptake, and that the rate of increase increases too, but we see no evidence that there is a sharp increase in derivative of the regression function for this example. In addition, the modelling machinery will produce a measure of uncertainty for the estimated threshold that is valid when the two-segment linear model is correct, but is misleading when the model is incorrect.

It is clear from Bennett's (1988) introduction that, from consultation with the collaborating experimental scientist, it was felt that there were strong *a priori* grounds for assuming the two-segment linear model. We believe that, as statisticians, we certainly do have a duty to build on prior information about the subject matter at hand when such is available and reliable. However, we should also be prepared to point out to the client when such prior "knowledge" simply isn't supported by the data. In this case, rather than simply provide the kinesiologist with an estimate of where the "best" place for an abrupt change seems to be, we should inquire about the degree of certainty behind the existence of an anerobic threshold, its ubiquity in different individuals, and its relationship with the expired ventilation measurement.

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REFERENCE

Bennett, G.W. (1988). Determination of anaerobic threshold. *Canad. J. Statist.*, 16, 307–316.

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